Investigation of Surface Roughness Effects on Adiabatic Wall Temperature

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Introduction

IN a flow in which viscous heating in the boundary layer is important, the adiabatic wall temperature is the local equilibrium temperature that a surface must attain in order to have zero heat transfer. Viscous heating is important in high-speed gas flows or the flow of high Prandtl number fluids at moderate speeds. Since the adiabatic wall (or recovery) temperature is the natural reference temperature for convective heat-transfer processes, heat-transfer data in the supersonic and hypersonic regimes are often presented as Stanton numbers, which are defined on the basis of the adiabatic wall temperature. Recent work by the authors strongly suggests that surface roughness can have a significant effect on the adiabatic wall temperature. This Note describes the results of a study addressing this effect.

Smooth Wall

The adiabatic wall temperature for a smooth wall is the surface temperature that results in the adiabatic wall condition

$$\left(\frac{\partial T}{\partial y}\right)_w = 0 \tag{1}$$

In computer codes that numerically solve the boundary-layer equations, Eq. (1) is easily implemented as a boundary condition and the adiabatic wall temperature is computed directly. Prior to the general availability of boundary-layer computer codes, the adiabatic wall temperature was typically calculated in terms of a recovery factor r. The recovery factor is used to relate the freestream static temperature T_∞ and Mach number M_∞ to the adiabatic wall temperature T_{aw}, the usual form being

$$T_{aw} = T_\infty\left(1 + r\frac{\gamma - 1}{2}M_\infty^2\right) \tag{2}$$

Generally, the recovery factor is taken as constant, with a value in laminar flow of $Pr^{1/2}$ and with a value in turbulent flow of $Pr^{1/3}$.

As a number of authors (e.g., Refs. 1-3) point out, these so-called constant recovery factors are not really constant, but are mildly dependent on the local flow parameters and the Reynolds number. To the authors' knowledge, no mention has ever been made of possible surface roughness effects on the recovery factor.

Rough Wall

The smooth wall definition of adiabatic wall temperature as given in Eq. (1) is not sufficient to define a rough-wall adiabatic temperature since it does not take into account what happens over the surfaces of the roughness elements. The adiabatic wall condition for a rough surface cannot be obtained by specifying that the temperature gradient normal to the surface be everywhere zero, since the fluid temperature profile in the vicinity of the roughness elements is established by an energy balance and cannot be constrained to satisfy everywhere on the surface the zero temperature gradient condition. As shown schematically in Fig. 1, roughness elements protrude into the boundary layer and expose the elements' surfaces to different temperatures at different locations above the base of the rough wall. The temperature profile depends on a local balance between viscous dissipation, convection, and diffusion. It does not seem possible to specify intuitively the effects of surface roughness on the balance of viscous dissipation, convection, and diffusion; however, each of these is affected in magnitude and distribution by the surface condition. Surface roughness can, therefore, be expected to have some effect on adiabatic wall temperature.

Thus, in formulating the concept of adiabatic wall temperature for a rough wall, both the smooth portion of the wall and the roughness elements must be considered. The adiabatic wall condition for a rough wall occurs when the net heat transfer between the fluid and the smooth and rough portions of the wall is zero. This condition is illustrated in Fig. 1, where a typical variation of temperature with distance above the wall is indicated. The temperature of both the smooth portion of the wall and the roughness elements is assumed to be the same. If the Biot number of a roughness element is much less than unity, then the resistance to conduction within the element is much less than the resistance to convection across the surface, and the assumption of a constant element temperature is viable. Under the conditions investigated in this study, roughness element Biot numbers would be much less

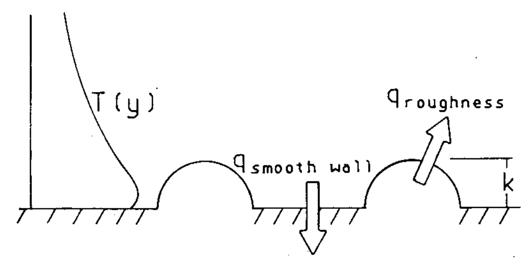

Fig. 1 Adiabatic wall condition for a rough wall.

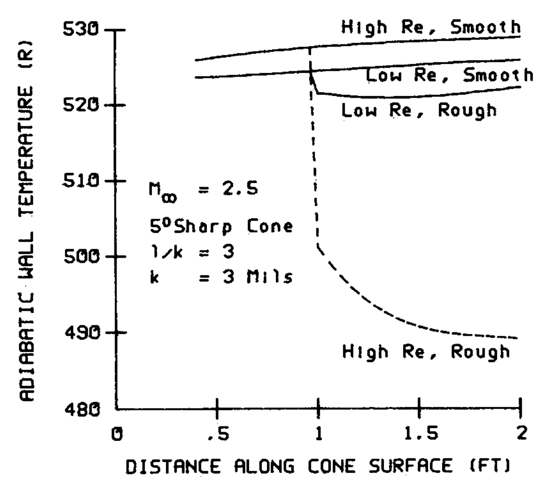

Fig. 2 Adiabatic wall temperature variation with Reynolds number ($1\times10^6 < Re_\infty/\text{ft} < 10\times10^6$), roughness begins at 1.0 ft.

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than 1 for any material having a thermal conductivity of order 0.10 Btu/h·ft·°R or greater; thus, most materials of interest would satisfy the assumption of equality of roughness element and smooth wall temperatures. For these cases, there is a net heat transfer from the roughness element to the fluid, because the local static temperature is less than the roughness element's surface temperature over most of the roughness height. If the wall is to be adiabatic, then heat must be transferred from the fluid to the smooth portion of the wall, giving rise to a nonzero temperature gradient at the surface of the smooth portion of the wall.

The authors are unaware of any existing adiabatic wall temperature data on a deterministic rough surface. In order to ascertain the magnitude of the expected change in adiabatic wall temperature due to surface roughness effects, the definition of adiabatic wall temperature described above was implemented in a turbulent, compressible boundary-layer computer program in which surface roughness effects were computed using the discrete element roughness model of Coleman et al.[4] The development and validation of this model are discussed in detail in Refs. 4 and 5. In the discrete element approach the rough surface is assumed to be composed of distinct roughness elements. This model is based on consideration of the physics of the interactions between the discrete roughness elements and the fluid and does not depend on the assumption of any sand-grain equivalent. The physical effects of the roughness elements on the fluid are modeled by considering the flow blockage, the local element/fluid heat transfer, and by postulating that the total force of the elements on the flow can be incorporated as form drag.

The apparent wall shear stress is composed of the viscous shear term acting over the portion of that wall not occupied by roughness elements plus the resultant form drag on the roughness elements. The apparent wall heat transfer is similarly the sum of the heat flux into the smooth portion of the wall plus the net heat diffusion into the roughness elements.

The computer program using the discrete element roughness model was modified to include the option of computing the adiabatic wall temperature over a rough surface. An iterative process was used to determine the surface temperature at which the net heat transfer from the roughness elements was equal in magnitude to the heat transfer to the smooth portion of the wall. This surface temperature is by definition the adiabatic wall temperature of a rough wall.

A study of roughness effects on adiabatic wall temperature was made in which Re_∞/ft, Mach number, roughness height, and roughness spacing ℓ were varied. The Reynolds numbers were $1\text{-}10 \times 10^6$/ft and freestream Mach numbers of 2.5 and 5.0 were considered. For appropriateness to wind tunnel test conditions, a 5 deg, 2 ft long cone (with the first 1 ft of length smooth) was used as the body shape. Hemispherical roughness elements spaced 3, 4, and 8 radii apart and with radii of 3 and 10 mils were selected as the roughness. These relatively small roughness geometries were chosen in order to ensure that the roughness heights were below the sonic line in the velocity profiles.

Complete results of this study are reported in Ref. 6. Typical results for high and low Reynolds numbers for the $M_\infty = 2.5$ case are presented in Fig. 2. These show that the effect of surface roughness is to reduce the adiabatic wall temperature from the smooth-wall condition. This was something of a surprise since surface roughness typically increases the heat transfer over that for corresponding smooth wall condition. However, the authors feel that this typical increase in *heat transfer* is no basis on which to argue that the *adiabatic wall temperature* for a rough wall should be greater than for the corresponding smooth wall case. As discussed previously, the fluid temperature profile is determined by a local balance of viscous dissipation, convection, and diffusion; and all three of these are influenced by surface roughness.

The results presented herein are consistent within the framework of the roughness model of Coleman et al.[4] and Taylor et al.[5] and serve to demonstrate that surface roughness can alter the adiabatic wall temperature from the smooth-wall value. The results of the study at a Mach number of 5.0 are qualitatively similar to the results at a Mach number of 2.5 and are reported in Ref. 6.

For the high Reynolds number results shown in Fig. 2, the recovery factors are about 0.8, a substantial reduction from the 0.89 predicted by using $Pr^{1/3}$. Burmeister[2] shows smooth-wall recovery factors for air in turbulent flow to be in the range 0.85-0.93 and in laminar flow to be in the range 0.82-0.87. The range of rough-wall turbulent recovery factors found in this study[6] is as large as the range shown by the smooth-wall data for *both* the laminar and turbulent regimes.

Conclusion

The results of the numerical study indicate that surface roughness can have an important effect on adiabatic wall temperature. In particular, surface roughness characteristics, freestream Reynolds number, and freestream Mach number all appear to be important parameters in determining the deviation of the adiabatic wall temperature from the smooth wall value. The results of this study and the absence of experimental data that address the question of surface roughness effects on adiabatic wall temperature indicate a need for additional experimental and analytical work on this problem.

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Constant-Density Approximation to Taylor-Maccoll Solution

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Introduction

THE constant-density solution for steady conical flow in the hypersonic range was studied by several researchers, e.g., Feldman[1] and Hayes and Probstein.[2] In this Note, the Taylor-Maccoll equation is so arranged that the nonlinearities may be grouped into one term which is zero on the cone surface as well as at the oblique shock and that, by neglecting this term, the constant-density approximation may also be used in supersonic range with good accuracy.

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